The Role of Network Coding in Wireless Unicast

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Introduction

- Wireline: Some well understood facts:
  - Unicast: Information flow = Commodity Flow
  - Role of network coding strictly for more general settings.

- Wireless: Network Coding seems to have a role even for unicast, which is what we would like to explore further.

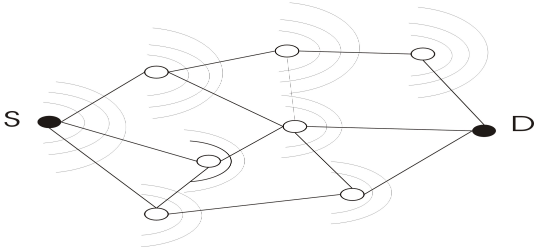
- Two issues distinguish wireless from a wireline setting:
  - Interference: A challenge to be overcome
  - Local Broadcast of the medium: An opportunity to be exploited ← *This will be the focus.*

§ Scheduling issues typically address interference by preventing interfering links operating together.

§ Given a schedule to work with, how do we exploit the local broadcast?

A Wireless Erasure Network Model

- Problem Model: Unicast, Lossy (erasure) Network



- $\chi(i, Z, t)$ - denotes success from i to Z at time t
- Capacity Constraint: $E[\chi(i, Z, t)] = c(i, Z)$
  - Rate at which exactly the nodes in Z receive transmission

  Example: $p(i, j) = \sum_{Z: j \in Z} c(i, Z)$

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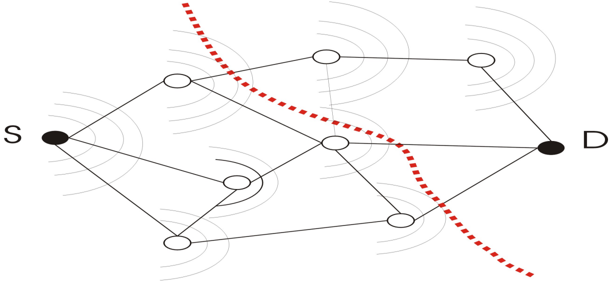


$$C(A) = \sum_{i \in A, Z \subseteq \mathcal{N}(i), Z \cap \bar{A} \neq \phi} c(i, Z)$$

Cut Value: The sum of the capacities of all hyperedges that have an overlap across the cut: i.e. rate at which at least one neighbor across the cut hears.

- Min Cut achieved with linear coding: [DGPHE06]

A general max flow formulation

$$\max \sum_{p \in P} x_p$$

(Max sum of flow along all paths $p \in P$)

Subject to:

$$\sum_{j:j \in Z} r(i,j,Z) \leq c(i,Z)$$

(Broadcast flow split among constituent edges)

$$\sum_{\{p \in P:(i,j) \in p\}} x_p \leq \sum_{\{Z:j \in Z\}} r(i,j,Z)$$

(Classical flow constraint for each edge , $(i,j) \in E$)

$$GMC = \min_{A,\bar{A}} \sum_{i \in A, Z \subseteq \mathcal{N}(i), Z \cap \bar{A} \neq \phi} c(i,Z)$$

Some Fundamental Observations

- Theorem: Max Flow is equal to GMC. Implies coding not necessary, if throughput is the sole concern.

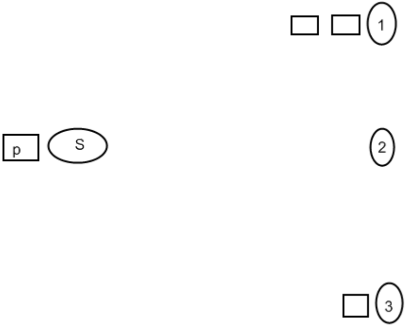
- Theorem: Backpressure scheme achieves GMC.

Remarks:

- The fact that coding is not necessary was also established previously by [Smith,Hassibi 08] without using the max flow interpretation.
- Backpressure schemes for this model were studied by [Neely 09], but they weren't related to the max flow min cut duality and consequently, the information theoretic min cut.
- Does coding have a role here beyond throughput?

Feedback Overhead for Routing policies

The Backpressure policy for wireless local broadcast:



Feedback Overhead for Routing policies

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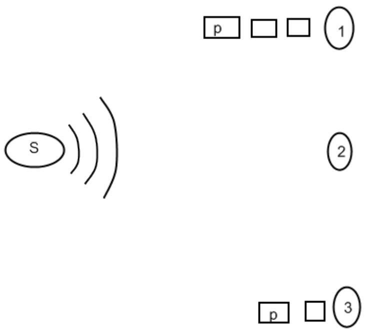

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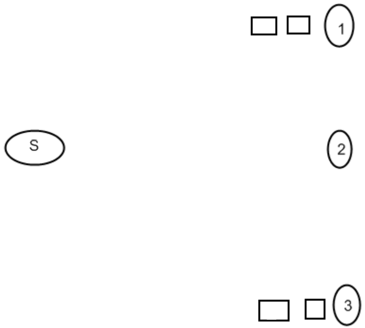


Feedback Overhead for Routing policies

The Backpressure policy for wireless local broadcast:

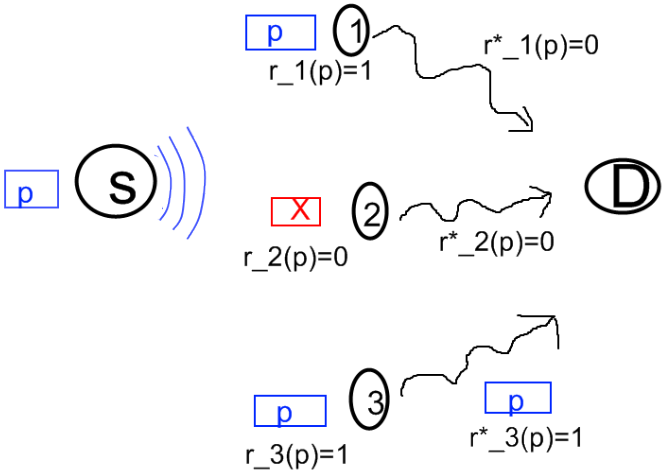
- Dynamic Routing Choices need to be made by *conferencing* among relays.
- Potential role of coding in solving this distributed synchronization problem.
- Perhaps there is some routing policy can avoid this overhead while still achieving a good throughput...

Feedback Independent Routing

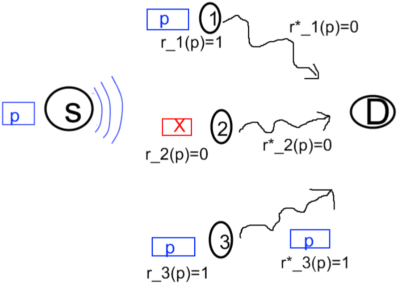
Key Issues:

- Characterize reasonable "Feedback constraints"
- What is the constrained capacity?
- Evaluate the merits of routing versus coding under such constraints.

Feedback Independent Routing



Feedback Independent Routing



FIR constraint for node S: For any disjoint sets of relays A, B, conditioned on $r_i(p) = 1 \ \ \forall i \in A$ we need: $\{r_i^*(p) : i \in A\}$ and $\{r_i(p) : i \in B\}$ are independent

- Informally: Each relay has to make distributed decisions on whether to forward a packet without consulting about the packet's receipt at other relays

Some schemes that satisfy FIR

- Send each packet to a specific relay. i.e. if a relay other than the intended one receives the packet, it discards the packet: used in practice, does not exploit local broadcast.

- Opposite extreme: *Flooding*. i.e., each relay keeps all the packets it receives and makes a random selection of a subset from it, as large as its rate to the destination can support.

- A generalization of the extremes: Tag a fraction $t(Z)$ of the packets with set Z. Upon receipt of a packet tagged with Z, a relay discards the packet unless it is a member of Z.

Characterizing the Capacity under FIR (C_{FIR})

A class of feasible schemes under FIR constraint:

$$\max \sum_{Z,Z' \subseteq [m] : Z \cap Z' \neq \phi} t(Z)c(Z')$$

$$\textit{Subject to}:$$

$$\sum_{Z \subseteq [m]} t(Z) \leq 1; \quad t(Z) \geq 0 \ \ \forall Z \subseteq [m]$$

$$p(S,i)\left(\sum_{Z \subseteq [m] : i \in Z} t(Z)\right) < p(i,D) \ \ \forall i \in [m]$$

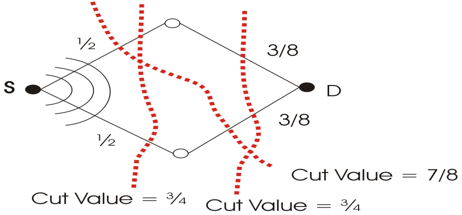
- Above LP represents the throughput of blind feedforward 'tagging schemes' where $t(Z)$ fraction of packets are tagged with destination Z.

Capacity under Feedback Independent Routing (C_{FIR})

- Theorem: Given any policy that satisfies *FIR*, there exists a blind tagging policy that matches its throughput.

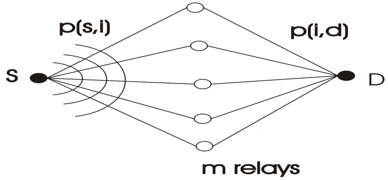
A simple application of the above theorem:

- C_{FIR} can be strictly less than GMC.



- $GMC = 3/4$, but $C_{FIR} = 5/8$ (by evaluating the LP)

Limitations of FIR



- An extreme case: $(p(S,i) = 1 \;\; \forall i \in [m])$ and $(p(i,D) = \frac{1}{m})$
- GMC is 1 and random linear coding can achieve this unit rate.
- Consider m packets in total received by all relays in m time slots.
- One expected onward transmission opportunity per relay per m total slots. Without coordination, expected number of distinct packets delivered $\approxeq m(1-(1-\frac{1}{m})^m) \approxeq 0.63m$ for large m.
- General case with asymmetry and arbitrary min cuts?

Applications of C_{FIR} characterization

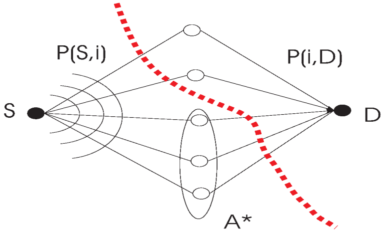
- Theorem: As long as the losses to relays are independent, for arbitrary configurations of rates and their corresponding min cut values, we have:

$$C_{FIR} \geq 1 - e^{-GMC} \geq 0.63$$

- In fact, as $GMC \to 0$, $\frac{C_{FIR}}{GMC} \geq \frac{1-e^{-GMC}}{GMC} \to 1$

- If the given network is lossy, extensive coordination using feedback or using coding can only give marginal relative benefit.

Independent Relay Losses



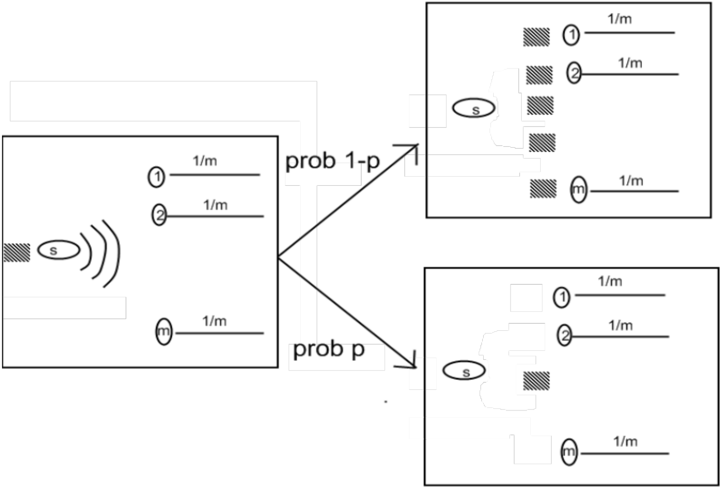
$$GMC = \min_{A \subseteq [m]} \left(1 - \prod_{i \in [m]/A} (1 - p(S,i)) + \sum_{i \in A} p(i,D) \right)$$

Flooding policy, $\mathcal{P}_f$: each relay selects a random subset to forward.

$$C(\mathcal{P}_F) = 1 - \prod_{i \in [m]} (1 - \min(p(S,i), p(i,D)))$$

Theorem: $C(\mathcal{P}_F) \geq 1 - e^{-GMC}$

Dependent Relay Losses: A special example



$GMC = 1$, but for appropriate p (say, $1 - \frac{1}{\sqrt{m}}$), C_{FIR} goes to 0.

Dependent Relay Losses

- $\frac{C_{FIR}}{GMC}$ not lower bounded by any positive quantity in general. Shown on the example network by evaluating a bound on the C_{FIR} based on its LP characterization.

- This bottleneck situation roughly represents the case where a large proportion of the packets at each relay are commonly received, but this set of commonly received packets turns out to be a small proportion of the overall set of packets in transit. Most of the relays end up wasting their resources on what are mostly duplicate transmissions.

- An Implication: Network coding is essential to overcome the distributed synchronization problem without extensive feedback signalling in the general case.

Conclusions

- Because of max flow min cut duality, dynamic opportunistic routing schemes can exploit local broadcast as effectively as network coding does.
- If relays have to make distributed dynamic routing decisions however, the throughput achieved by routing policies strictly decreases.
- The decrease in throughput is bounded if the link losses are all independent, but it could become arbitrarily bad when there are dependencies among link losses.
- The role of network coding for wireless unicast is to solve the distributed synchronization problem without extensive feedback, rather than as a way to increase the throughput by exploiting local broadcast.