Computing the Capacity Region of a Wireless Network

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INFOCOM 09

Network Capacity: Basic Problem

- Model of Wireless Network
  1. Directed Graph
  2. Protocol Model (i.e. no Coding in network, or any Information theoretic schemes, no exploiting of broadcast multipath, etc.)
  3. Interference constraints: Specified via link pair conflicts (Ex: k-hop, node exclusive, etc.)

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- *Given: (i) m S-D pairs; (ii) an $m-$dim rate vector,* $\mathbf{r}$
  Question: Does $\mathbf{r} \in \mathcal{F}$? (i.e., can we satisfy the following two constraints simultaneously?)
  1. m flows - i^{th} flow of value r_i between S-D pair i (routing)
  2. Sum of flows convex combination of non-conflicting link subsets. (scheduling)
     (i.e., is there a TDMA scheme which supports the sum of flows?)

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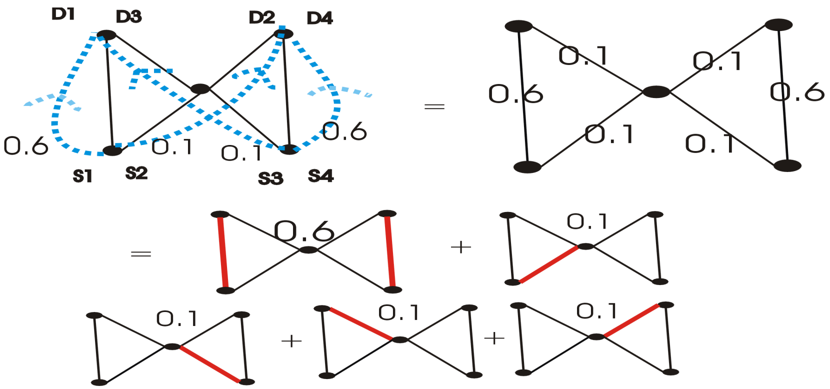

We say end to end rate vector, $(0.6, 0.1, 0.1, 0.6)$ is feasible

Background on rate feasibility

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- Given end-to-end rates and a flow routing decomposition, end-to-end feasibility reduces to link rate feasibility
- Our previous work on single hop: *"Feasible Rate Allocation in Wireless Networks", INFOCOM '08* provides approximate poly time oracles for link rate feasibility in restricted graphs.

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- Our Result: Polynomial algorithm for n^2 dimensional unicast capacity upto arbitrary accuracy when network graph allows for MWIS approximation (in many practical situations, it does)
  Algorithm declares in poly time:
  'YES', if $(1+2\epsilon)\mathbf{r} \in \mathcal{F}$
  'NO' if $(1-2\epsilon)\mathbf{r} \notin \mathcal{F}$

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  Recall: Markov Chain for i.i.d. packet arrivals stable under "MWIS routing/scheduling" iff $r \in \mathcal{F}$; *"Stability properties...", Tassiulas, Ephremides, TAC '92*

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- Caveat: Stability is only existential statement. But, how long do we need to observe queue lengths and how do we make the call on feasibility? (in poly time)

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- Key issues to be addressed:
  (i) Does *approximate* MWIS $\Rightarrow$ *approximate* stability?
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  - Assume $\mathbf{r}$ avoids an $\epsilon-$ boundary of the feasibility region and declare the right answer
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- Deterministic real valued queue length process $\Rightarrow$ explicit bounds

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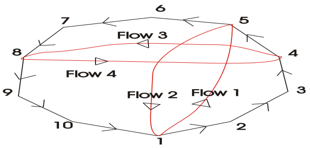
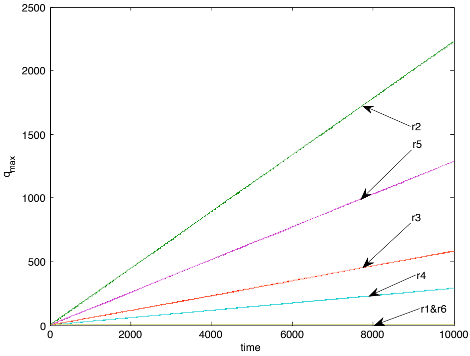
  - Decision in at most $\frac{p_1(n)p_2(n)}{\epsilon^3}$ time slots (*unless* $\mathbf{r}$ *falls in the* 2ϵ*-boundary*).
  - t time slots of $\epsilon/2$-MWIS $\Rightarrow$ Feasibility of $(1 \pm \epsilon(t))\mathbf{r}$ where $\epsilon(t) = 2\min\left(\sqrt{\frac{q^{max}(t)p_2(n)}{t}}, \frac{p_1(n)}{q^{max}(t)}\right)$ when $\epsilon < \epsilon(t) < 1/2$

    Assured that $\epsilon(t)$ *decreases below any* $\epsilon > 0$ *in poly time.*

Comments

- $p_1(n) = n^{7.5}$, $p_2(n) = n^2$, but the algorithm could be potentially more efficient in practice than the guarantees provided.





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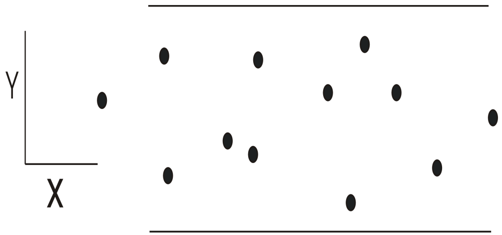
- Graphs with polynomial growth:
  1. Geometric random graph with $O(\log n)$ communication radius
  2. Arbitrary geographic graphs with bounded density and communication radius

A Special case with Exact algorithm



- Motivated by IVHS applications.

- Bounded radius of communication and interference.

- End-to-end rate feasibility can be posed as a polynomial LP by extending our previous work on link feasibility based on a fractional coloring algorithm.

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- Existing literature considers the problem of understanding the capacity for an ensemble of networks with a probabilistic distribution and as the network size goes to infinity (scaling laws).
- A byproduct of our work: Transient analysis of the approximate max weight scheduling algorithm for deterministic arrivals.